#### Examples: Divide 2. 5|35

1. 3|−15 3 ⋅ 2 = 3 ⋅ 8 = (−5) ⋅ 3 624= −15

3.

4.

5.. Let b) n ∈ ℕ and a, b ∈ ℤ. It applies that n|a and n|b. It follows that n|(a + b) and n|(a − Theorem: Divisor of Sums and Differences

also applies.

Proof:

) ⋅

4 ⋅ 2 = 7 ⋅ 2 = 14 = 6 + 3|9 8 3 ⋅ 3 ⋅ 2+3

m|(b − a)Example: Congruent moduloLet a, b ∈ ℤ applies. and let a ≡ b mod mm ∈ ℕ. We write is read “a ≡ b mod ma is congruent to , if m divides the number b modulo m.” b − a, i.e., if

Definition: Equivalence Relation2.3.4.1. It applies that It applies that It applies that It applies that −85221 ≡ −19 mod ≡ 12 mod ≡ 0 mod ≡ 4 mod 572, because 4, because 572|(12 − |(0 |(4 4|(−19 − − 21)− (−82) because 5)) because 2 ⋅ 5 = −36 ⋅ 2 = ⋅ 7 10= −21.⋅12.

following properties apply to all An equivalence relation is therefore a set whose elements must fulfill certain properties.Let 1.2.3. MReflexivity: Symmetry: If Transitivity: If be a set. A subset (x, x) ∈ R((x, y) ∈ Rx, y) ∈ RR ⊆ M × M, then and x, y, z ∈ M((y, z) ∈ Ry, x) ∈ R is called an equivalence relation to :, then also applies (x, z) ∈ R also applies M if the three

Often,reads this “cates to which quantity the equivalence relation refers.If R is an equivalence relation on instead of x is equivalent to x ∼R y, one simply writes y.” M and (x, y) ∈ Rx ∼ y in abbreviated form, if the context indi-, then we write x ∼R y for it. One a ∼Example: Equivalence relationsn b is an equivalence relation to ℤ:

a) ∼ , then n| relation If Definition: Equivalence Classa ∼∼( n b∼, then n is therefore also called congruent modulo equivalence relation.□n|(a − b) applies. This is equivalent to n|(a − b) (a − b) + (b − c) = a − ca ≡ b mod n. The equivalencen|(a − b) +n|(a − c)n|−(a − b) b)

b − c) then is c)

Let MFrom be a set and x, y ∈ LR be an equivalence relation to Rx ∼ y y ∈ L M. A non-empty subset L M L ⊆ M is called equivalence class with respect to if the following two properties apply:

it follows that .

If x ∈ L and y ∈ M, and if x ∼ y, then .

equivalence class L must be equivalent to each other in pairs. Property 2 states that every An equivalence class thus denotes a non-empty subset of a set , with respect to which

element of M which is equivalent to an element of equivalence class L must also be an we have formed an equivalence relation. Property 1 states that all elements of such an

element of this equivalence class.